GENERATION OF PLASMA WAVES BY THICK TARGET ELECTRON BEAMS AND THE EXPECTED RADIATION SIGNATURE

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ABSTRACT

The production of plasma waves by a nonthermal beam of high energy electrons injected into a background thermal plasma is investigated. The coupled kinetic equations for the plasma wave and particle distributions is used to place an upper bound on the energy density and spectrum of the plasma waves generated by this process. The situation of an inhomogeneous electron beam is considered which enables us to clarify some of the ambiguities which arise when one trys to treat the homogeneous situation.

It is shown that the wave-particle interactions have a significant, but not dominant, effect on the overall distribution of the electrons and that it is unlikely that such effects can be discerned in the observed bremsstrahlung or synchrotron radiation of the nonthermal electrons. Although a significant fraction of the nonthermal electron energy is transferred to plasma waves, the wave energy density is not very high because of their rapid attenuation by the thermal electrons. The subsequent conversion of the wave energy to transverse electromagnetic radiation is also discussed briefly. It is shown that the level of radiation produced is much less than the estimates of Zaitsev and Kaplan (1968) or Emslie and Smith (1984) and probably insignificant compared to direct synchrotron radiation by the nonthermal electrons.

I. INTRODUCTION

Nonthermal electrons with energies far exceeding the energies of the particles in a background thermal plasma are responsible for generation of electromagnetic radiation in many astrophysical situations. The superposition of the two distributions can give rise to a positive slope in the overall electron velocity distribution ($\frac{\partial f}{\partial v} > 0$). It is well known that such a distribution is unstable (two stream or bump-on-tail instability; cf., e.g. Krall and Trivelpiece 1973) and can generate plasma waves and a plateau in the overall electron velocity distribution ($\frac{\partial f}{\partial v} \approx 0$) within a short time, τ_{pl}, which is on the order of the inverse of the plasma frequency. Such a turbulence (we will use plasma waves and turbulence interchangeably) is limited to a small region of linear size $< c\tau_{pl}$ around the acceleration or injection site of the nonthermal electrons. However, as the electrons propagate their interaction with the background plasma can generate more turbulence throughout the plasma if the interaction cross section decreases with increasing energy or velocity. Coulomb collisions have this property and will tend to erode away the plateau continuously giving rise to a positive slope in the velocity distribution and therefore to plasma turbulence. Sometime ago, Zeitsev and Kaplan (1968) suggested that the conversion of the plasma waves generated by this process into transverse electromagnetic radiation could be more important than the synchrotron mechanism in generating radio and microwave radiation in cosmic sources.

This mechanism clearly requires a plasma with a high particle density, n, so that the plasma frequency $\nu_p = 9 \times 10^3 \mathrm{Hz}\sqrt{n/\mathrm{cm}^{-3}}$ is in the GHz range and that the Coulomb collisions are the dominant dissipation process. One astrophysical situation where this condition is believed to be present is during the impulsive phase of a solar flare. The electrons responsible for most of the radiation during an impulsive phase are most probably accelerated somewhere high up in the corona and (if not ultrarelativistic) loose most of their energy through Coulomb collisions at densities exceeding $10^{10}\mathrm{cm}^{-3}$. Thus plasma turbulence will be generated and the resultant electromagnetic radiation will be in the microwave range. Recently, Emslie and Smith (1984), using the estimation of the level of the plasma waves prescribed by Zeitsev and Kaplan (1968), evaluated the expected microwave flux during a typical flare. They obtained a high flux and concluded that this constitutes a strong constraint on the nonthermal thick target model for the impulsive phase of a flare.

As we shall see in § III both of the above mentioned works overestimate the level of contribution of this process to the observed radiation. There we will compare the direct radiation production processes by the nonthermal electrons with the radiation production by waves. Before this, in the next section we examine the coupled particle, Langmuir wave kinetic equations and present a more rigorous derivation of the level of plasma wave energy density and its distribution. In particular, we solve these equations for the more realistic inhomogeneous condition. A brief summary is presented in § IV.

II. LEVEL OF PLASMA TURBULANCE

A. *General Equations*

The astrophysical situation under consideration is the generation of Langmuir waves by a suprathermal beam of electrons passing through a Maxwellian plasma. The classical form of the kinetic equations, the quasi-linear equations, describe the evolution of the distributions of particles and waves due to the emission and absorption of the waves by the particles (e.g. Melrose 1980). In most astrophysical conditions, the geometry of the nonthermal source is determined by the magnetic field since it constrains the orbits of charged particles. The motion of an electron with Lorentz factor γ and velocity βc in a uniform magnetic field of strength B consists of spiralling at the gyrofrequency $\nu_B = 2.8 \times 10^8 \mathrm{Hz}\ (B/100\mathrm{G})$ with a gyroradius $r_B \approx 10\beta\gamma(100\mathrm{G}/B)\mathrm{cm}$, while the velocity parallel to the field lines remains constant. The gyroradius in general is much smaller than the spatial variations in the plasma. Therefore diffusion of electrons across field lines can be neglected.

Then the only variation in the distribution function is along the magnetic field lines. As mentioned above the process under consideration is important only at high plasma densities so that the plasma frequency is large. We shall assume that the plasma frequency is larger than the gyrofrequency, which means $n > 10^9\,\mathrm{cm}^{-3}(B/100\mathrm{G})^2$, and ignore the effects of the magnetic field on the equations. This does not invalidate the above assumption about the particle being tied down to the field lines. Although $\nu_p > \nu_B$ the magnetic field energy could be larger than the plasma energy density: $\beta_p \equiv 8\pi n\kappa T/B^2 = 2(\nu_p/\nu_B)^2(\kappa T/mc^2) < 1$ for a nonrelativistic plasma. In the case of solar flares $\nu_p \approx \nu_B$ so a more complete analysis is required. We believe that the results presented below, found by assuming $\nu_B \ll \nu_p$, would give a rough but realistic estimate of the conditions even when $\nu_p \approx \nu_B$. We shall therefore ignore the effects of the magnetic field and treat the nonrelativistic problem. The effects of the magnetic field and the full relativistic analysis will be described in a future work.

Under these conditions the emission of Langmuir waves is via the Cerenkov process. Then, as shown in the appendix, for a well collimated beam of electrons the electron and wave distributions can be considered to be functions of only the velocity parallel to the magnetic field. For a nonrelativistic beam of electrons the equations describing the evolution of the distributions of the particles and waves are (see equations $A10$, $A26$, and $A31$ in the appendix)

$$\frac{df}{dt} = \nu_p \frac{\partial}{\partial v}\left(\frac{2\pi^2 v_T^2}{n_T} v W \frac{\partial f}{\partial v} + \frac{2\nu(v_T)}{\nu_p \ln\Lambda}\frac{v_T^3}{v^2}\ln(v/v_T) f + \frac{\nu(v)}{\nu_p} v f\right), \tag{1}$$

$$\frac{dW}{dt} = \nu_p \left(\frac{2\pi^2}{n_T} v^2 W \frac{\partial f}{\partial v} + \frac{2\nu(v_T)}{\nu_p \ln\Lambda}\frac{v_T}{v}\ln(v/v_T) f - \frac{\gamma_{coll}}{\nu_p} W\right) . \tag{2}$$

Here $f(x,v,t)$ and $W(x,v,t)$ are the distributions of the electrons and waves as a function of distance x and velocity v along the field line defined such that the electron density n and the Langmuir wave energy density ε_w are

$$n(x,t) = \int f(x,v,t)\, dv\,, \quad \varepsilon_w(x,t) = \kappa T \int W(x,v,t)\, dv\,. \tag{3}$$

$W(v)$ is related to the number density $N(k)$ of waves with wave vector $k = v/2\pi\nu_p$ by $W(v)dv = (\frac{h\nu_p}{\kappa T})N(k)dk$. The first term on the right hand sides of equations (1) and (2) accounts for wave particle interactions and the second term with coefficient $\ln(v/v_T)$, where $v_T = (\kappa T/m)^{1/2}$ is the mean thermal velocity of the electrons in the background plasma, describes the effect of spontaneous emission of waves. These equations are essentially the same used in the numerical work done by Takakura and Shibahashi (1976), except we have included the Coulomb interactions between the nonthermal and background electrons and collisional damping of the waves.

The effect of collisions on the electron distribution can be found using a Fokker-Plank analysis. In the one dimensional treatment this gives rise to a term $\frac{\delta f}{\delta t}|_{coll} = \frac{\partial}{\partial v}(\nu(v)vf(v))$, which is represented by the last term in equation (1). The collisional damping of the waves is accounted for by the damping rate $\gamma_{coll}W$, the last term in equation (2). As shown in the appendix these equations are an excellent approximation to the more realistic three dimensional situation for moderate degrees of beaming of the nonthermal electrons (velocity dispersion perpendicular to the field less than the velocity along the field).

Although equations (1) and (2) are a full mathematical representation of our system, we wish to work with a different, but equivalent set. By using equation (2) to elimate the term involving the product of W and $\frac{\partial f}{\partial v}$ in equation (1) we find

$$\frac{d}{dt}\left[f(x,v,t) - \frac{\partial}{\partial v}\left(\frac{v_T^2}{v}W(x,v,t)\right)\right] = \frac{\partial}{\partial v}\left(\frac{\nu(v_T)v_T^2}{2v}W(x,v,t) + \nu(v)vf(x,v,t)\right) . \quad (4)$$

This replaces equation (1) in our description.

When treating the situation of a suprathermal electron beam passing through a background plasma, it is useful to separate the electron distribution into two parts. So we let $f(x,v,t) = f_s(x,v,t) + f_T(v)$, where $f_s(x,v,t)$ is the contribution from the suprathermal electrons and $f_T(v)$ is the distribution of the background (Maxwellian) plasma. Similarly we let $W(x,v,t) = W_s(x,v,t) + W_T(v)$. The background plasma is assumed to satisfy the homogeneous form of the above equations ($\frac{df_T}{dt} = 0$ and $\frac{dW_T}{dt} = 0$).

1. *Background Distributions*

The steady state thermal equilibrium condition leads to the background electron distribution (one dimensional)

$$f_T(v) = \frac{n_T}{\sqrt{2\pi} v_T} \exp(-v^2/2v_T^2) \ , \tag{5}$$

which is the Maxwellian distribution integrated over the perpendicular velocity components. The background wave energy density can be obtained by substitution of this in equation (2). We note first that

$$\gamma_{coll}/\nu_p \sim \nu(v_T)/\nu_p \sim \frac{1}{n_T \lambda_D^3} \sim \frac{1}{N_D} \ll 1 \ , \tag{6}$$

where $\lambda_D = v_T/2\pi\nu_p$ is the electron Debye length and $N_D = \frac{4\pi}{3} n_T \lambda_D^3 \gg 1$ is the total number of electrons within a Debye sphere. Furthermore since the average value of $(v^2 \frac{\partial f_T}{\partial v}) \approx n_T$ it is clear that the last term in equation (2) is negligible for the background conditions. So that $vW \sim \lambda_D^{-3}/(-\frac{\partial \ln f_T}{\partial \ln v}) \sim \lambda_D^{-3}$ and that the backgroung wave energy density is

$$\varepsilon_T \approx \kappa T \lambda_D^{-3} \ . \tag{7}$$

We shall see that the opposite is true for the suprathermal electrons in that the spontaneous emission term is negligible and $1 \gg \frac{\partial \ln f_T}{\partial \ln v} \gtrsim 0$.

2. *Equation for Suprathermal Particles*

If the density of the high energy electrons $n_s = \int f_s \, dv \ll n_T$ then for the streaming electrons the collision frequency is determined by the density of the background electrons and the damping rate of the waves produced by the stream (which have wavelengths $> \lambda_D$) are, respectively

$$\nu(v) = 2\pi c r_o^2 n_T \ln\Lambda (c/v)^3 \ , \ \ \gamma_{coll} = \nu(v_T)/2 \ , \tag{8}$$

where $r_o = 2.8 \times 10^{-13}$cm is the classical radius of the electron and $\ln\Lambda \approx 20$ is the Coulomb logarithm. For parameters appropriate for a flare

$$\zeta \equiv \nu(v_T)/\nu_p = 1.5 \times 10^{-6} \left(\frac{\ln\Lambda}{20}\right) \left(\frac{n_T}{10^{10}\mathrm{cm}^{-3}}\right)^{1/2} \left(\frac{10^6}{T}\right)^{3/2} \ . \tag{9}$$

Note that ζ^{-1} is of the order of N_D the number of electrons in a Debye sphere which must be much greater than unity for the validity of all our equations.

Using these in equation (4) the distribution of the suprathermal electrons satisfies

$$\frac{d}{dt}\left[f_s(x,v,t)-\frac{\partial}{\partial v}\left(\frac{v_T^2}{v}W_s(x,v,t)\right)\right]=\nu_p\zeta v_T^2\frac{\partial}{\partial v}\left(\frac{W_s(x,v,t)}{2v}+\frac{v_T f_s(x,v,t)}{v^2}\right) \ . \quad (10)$$

If the velocity of suprathermal electrons is much greater than the thermal velocity v_T the cross terms $W_s\frac{\partial f_T}{\partial v}$ and $W_T\frac{\partial f_s}{\partial v}$ in equation (2) can be ingnored so that we have

$$\frac{dW_s}{dt}=\nu_p\left(\frac{2\pi^2}{n_T}v^2W_s\frac{\partial f_s}{\partial v}+(\frac{2\zeta}{\ln\Lambda})(\frac{v_T}{v})\ln(v/v_T)f_s-\frac{\zeta}{2}W_s\right) \ . \quad (11)$$

The derivative $\frac{d}{dt}$ in all of the above equations is the total time derivative so that $\frac{d}{dt}=\frac{\partial}{\partial t}+v\frac{\partial}{\partial x}$ for the distribution f and $\frac{d}{dt}=\frac{\partial}{\partial t}+v_{gr}\frac{\partial}{\partial x}-\frac{d\omega_p}{dx}\frac{\partial}{\partial k}$ for W, where $\omega_p=2\pi\nu_p$ and v_{gr} is the group velocity of the waves. If the acceleration and injection time scales of the electrons are much longer than the relaxation time scale then the explicit time dependence can be ignored. This will be true if the injection time $\tau_{inj}\gg\max(\nu(v)^{-1},\nu_p^{-1})$. For ν_p in the microwave range (10^{10} GHz) τ_{inj} will clearly exceed ν_p^{-1} which is the characteristic time for plateau formation in the quasi-linear analysis (Grognard 1975). However, for most astrophysical situations $\zeta\ll 1$ so that we require $\tau_{inj}\gg 1/\nu(v)$. This also is true in most astrophysical situations, in particular for the impulsive phase of solar flares except for millisecond time variability. Thus for observations with time resolution of a second or larger we can ignore the implicit time variation and look for solutions with $\frac{\partial f}{\partial t}=0$ and $\frac{\partial W}{\partial t}=0$.

3. *Effects of Collisions*

Since the waves are generated by collisional errosion of the plateau we reproduce here some known effects of the collision term on the distribution of stream electrons. This will be useful in our treatment of the full equations.

Ignoring the wave-particle processes, the steady state electron distribution of the suprathermal electrons is given by

$$v \frac{\partial f_{coll}(x,v)}{\partial x} = \frac{\partial}{\partial v}\Big(\nu(v) v f_{coll}(x,v) \Big) \ . \tag{12}$$

We use f_{coll} to indicate that only collisions are included. The well-known solution of this equation is

$$f_{coll}(x,v) = [1 + x/\lambda(v)]^{-1/2} \ f_{in}(v[1 + x/\lambda(v)]^{1/4}) \ , \tag{13}$$

where $f_{in}(v) = f_s(x = 0, v)$ is the electron distribution at the point of injection, and $\lambda(v)$, the collisional stopping distance in the nonrelativistic limit, is

$$\lambda(v) = \frac{v}{4\nu(v)} = 3.1 \times 10^{10} \left(\frac{v}{10^{10}\,\mathrm{cm/s}}\right)^4 \left(\frac{10^{10}\,\mathrm{cm}^{-3}}{n_T}\right) \left(\frac{20}{\ln\Lambda}\right) \mathrm{cm} \ . \tag{14}$$

This means that the slower particles are removed at smaller depths than the faster particles therefore producing spatial variations in the electron beam. At any fixed position the overall electron distribution will have a " bump-on-tail" distribution.

To illustrate this effect we take as our initial injected distribution a distribution which is stable to Langmuir wave growth

$$f_{in}(v) = \frac{2\Gamma(\delta)}{\sqrt{\pi}\Gamma(\delta - \frac{1}{2})} \frac{n_s/v_s}{[1 + (v/v_s)^2]^\delta} \ , \quad n_s = \int f_{in}(v)\, dv \ , \tag{15}$$

where Γ's are the well known gamma functions. Substituting this in equation (13) we obtain $f_{coll}(x,v)$ and the distribution $f(x,v) = f_{coll}(x,v) + f_T(v)$ for all plasma electrons, which is plotted as a function of velocity at different positions in figure (1). In the velocity range where $\frac{\partial f}{\partial v} > 0$ the distribution is unstable and gives rise to Langmuir waves with phase velocities within the same range. We see that for values of the densities and velocities typical for solar flares this velocity range is quite large and increases with depth although decreasing in magnitude.

B. *Spatially Homogeneous Case*

Previous investigators of this problem (Zaitsev and Kaplan 1968) have assumed not only steady state but also spatial homogeneity. We believe this to be an unrealistic assumption. To realize this situation the high energy particles must be injected uniformily thoughout a uniform plasma of dimension much larger than their collisional mean free path ($\lambda(v) \sim v/\nu(v)$) and there must exist a sink to deposit the injected energy at a rate equal to that of injection. This is not the condition in most astrophysical situations and it clearly is not the case for a solar flare where it is believed that the particles are injected at the top of a loop in the corona and have mean free paths comparable to the length of the loop. However, to clarify the origin of the previous order of magnitude estimates and correct some errors we first treat this problem using the full equations (equations 10 and 11). We shall see in the next part that the assumption of spatial homogeneity is not even necessary and that our analysis of the more realistic situation described there resolves some of the ambiguities that we shall find with the present assumptions.

With the assumption of spatial homogeneity and if we ignore the spontaneous emission term (see below for justification) the equations simplify to

$$0 = \frac{d}{dv}\left(\frac{1}{2v}W_s(v) + \frac{v_T}{v^2}f_s(v)\right) \ , \tag{16}$$

$$\frac{df_s}{dv} = \frac{n_T \zeta}{4\pi^2 v^2} \ . \tag{17}$$

These equations are presumeably valid in the velocity range $v_c \lesssim v \lesssim v_s$, where v_s is the characteristic velocity of the injected nonthermal particles and v_c is the lowest phase velocity where significant levels of plasma waves exist, which presumeably occurs at the velocity where the contributions to the total distribution from the nonthermal and thermal electrons are comparable. As evident from the dashed line on the left in figure (1) for $n_s \ll n_T$ and $v_s \gg v_T$ the critical velocity v_c will be a few times v_T. We rewrite equation (17) to obtain the logarithmic derrivative of the distribution

$$0 < \frac{d\ln f_s}{d\ln v} \approx \eta \equiv \frac{n_T}{n_s}\frac{\zeta}{4\pi^2} \ . \tag{18}$$

Here we have assumed that the total nonthermal particle density $n_s \approx v_s f_s$. Since $\zeta \ll 1$, eventhough $n_T/n_s \gg 1$, in general, $\eta < 1$ and a plateau is formed. For example, if we use typical values for solar conditions: $n_T/n_s < 10^6$, $v_s/v_T = 25$, $n_T = 10^{10}\mathrm{cm}^{-3}$, and $T = 10^6\mathrm{K}$, then $\eta < 4 \times 10^{-2} \ll 1$.

We can also make an order of magnitude estimate of the wave energy density by equating the two terms in equation (16). (Note, however that both of these terms are positive quantities). This gives $W_s(v) \approx 2(v_T/v)f_s(v)$ so that the wave energy density $\varepsilon_w \approx 2\kappa T n_s(v_T/v_s)$. This is essentially the result of Zaitsev and Kaplan (1968), if one sets their $\Delta k/k$, where k is a mean value for the wave vector and Δk is the range of the wave vectors, equal to unity. This is also the relation used by Emslie and Smith (1984). However, they overestimate the wave energy density by a factor of $\sim v_s/2v_T$ because they set $\Delta k = \pi\nu_p/v_T$. We shall see below that this is incorrect because although k_{max} for the waves is near $2\pi\nu_p/v_c \sim 2\pi\nu_p/v_T$ most of the waves have $k \sim 2\pi\nu_p/v_s$ so that $\Delta k/k \sim 1$ is the correct value. Note that with the above estimates for W_s and $\frac{df_s}{dv}$ one can show that the spontaneous emission term which we ignored is smaller than the other two terms by a factor of the order of the Coulomb logarithm.

For a more exact treatment equations (16) and (17) must be integrated. Integration of equation (17) leads to

$$f_s(v) = \frac{n_s}{v_s}\left(\xi - \eta\frac{v_s}{v}\right) , \quad v_c < v < v_s \ , \tag{19}$$

where we have set the constant of integration equal to $\frac{n_s}{v_s}\xi$ with n_s as the number of the injected high energy electrons (or $n_s v_s$ as the flux of injected electrons). Since these electrons loose energy both to Langmuir waves and to the background particles through collisions the equilibrium density of particles in the plateau is in general less than n_s. For $v_s/v_c \gg 1$ and $\eta \leq \xi\frac{v_c}{v_s} \ll 1$ from equation (19) we have $\int_{v_c}^{v_s} f_s \, dv \approx n_s\xi$ with $\xi < 1$.

Integration of equation (16) gives

$$W_s(v) = \frac{2v_T}{v_c}\left(\frac{v}{v_c}f_s(v_c) - \frac{v_c}{v}f_s(v)\right) \ , \tag{20}$$

where we have chosen the constant of integration so that $W_s(v_c) = 0$ as we do not expect excitation of waves with phase velocities below the plateau. If we assume that the plateau extends from v_c to v_s so that $f_s(v_c) = f_s(v_s) = \xi \frac{n_s}{v_s}$ (this means that $\eta \ll \xi v_c/v_s$) then

$$W_s(v) = \frac{2\xi n_s v_T}{v_c v_s}\left(\frac{v}{v_c} - \frac{v_c}{v}\right) . \tag{21}$$

From this we obtain the energy density in plasma waves

$$\varepsilon_w = \kappa T \int_{v_c}^{v_s} W_s(v)dv \approx \xi n_s \kappa T(v_T/v_s)\left[(v_s/v_c)^2 - 1 - 2\ln(v_s/v_c)\right] , \tag{22}$$

which for $v_s \gg v_c$ becomes

$$\varepsilon_w \approx \xi n_s \kappa T(v_T/v_s)(v_s/v_c)^2 . \tag{23}$$

We can thus estimate ε_w using the full equations, but at this point we do not know the value of ξ or v_c (i.e. the values of the two integration constants). This of course can be done with solution of the complete set of equations where both the background plasma density and temperature and the distributions of suprathermal particles are calculated self-consistently. In the inhomogeneous situation described below, we relate the energy density in plasma waves directly to the distribution of the injected beam. There we will find an upper limit $\xi \lesssim (v_c/v_s)^2$.

C. *Spatially Inhomogeneous Case*

The assumption of a spatially homogeneous electron flux in part B allowed us to omit the advective terms in the equations (1) and (2). As mentioned above, this is valid provided that the scale of spatial variations is much larger than the characteristic wave growth time multiplied by the velocity of the electron beam. This places a significant constraint on the size of the injection region. However, in most situations encountered in astrophysics, and in particular for a solar flare, the size of the injection region is much smaller than the collisional mean free path of the electrons and the advective terms cannot be neglected.

While a numerical solution is needed to solve the coupled inhomogeneous equations, we now outline a procedure for setting an upper limit on the plasma wave energy density.

To simplify, we integrate equations (10) and (11) along the space coordinate to obtain equations relating the integrated electron and wave distributions. In fact, for comparison with observations which do not resolve the source region, the integrated values are the appropriate quantities. Integration of equation (10) gives

$$-vf_{in}(v) = \zeta \nu_p v_T^2 \frac{d}{dv}\left(\frac{\tilde{W}_s(v)}{2v} + \frac{v_T^2 \tilde{f}_s(v)}{v}\right) , \tag{24}$$

where we have defined the integrated quantities $\tilde{\varphi}(v) \equiv \int_0^\infty \varphi(x,v)dx$. Here we have used the boundary conditions $W_s(0,v) = W_s(\infty,v) = 0$ for the distribution of waves and $f_s(\infty,v) = 0$ for the electron distribution.

A couple of remarks are in order here. Equation (24) is correct if the background plasma density and temperature, on which the collision frequency depends, are constant or the scale of their variation is larger than the stopping length λ defined in equation (14): $\frac{d\ln n_T}{dx}$, $\frac{d\ln T}{dx} \ll 1/\lambda(v)$. Otherwise $\nu(v_T)$, v_T and $\nu(v)$ in (24) must be interpreted as mean values throughout the relevant region. Also the assumption about the boundary conditions on waves is not necessary. This is because for the steady state case $\frac{dW}{dt} = v_{gr}\frac{\partial W}{\partial x} - \frac{d\omega_p}{dx}\frac{\partial W}{\partial k}$. If the background is nearly homogeneous ($\frac{d\omega_p}{dx} \ll \omega_p/\lambda(v)$) then $\frac{dW}{dt} = v_{gr}\frac{\partial W}{\partial x}$, where $v_{gr} = 3v_T^2/v_s$ is the group velocity of the waves. The ratio of this term to $\nu(v_T)W$, the dominant term in equation (11), is $\sim (v_T/v_s)^2 \ll 1$. Thus, in general, $\frac{dW}{dt}$ can be set equal to zero which also means that if as before we ignore the spontaneous emission terms we obtain equation (17) for the particle distribution throughout the whole region. So, if the background plasma is homogeneous, but the injection may still be confined to a small region so that the beam will be inhomogeneous, we have

$$\frac{\partial f_s}{\partial v} = \frac{n_T \zeta}{4\pi^2 v} . \tag{25}$$

Integrating equation (24) and using the fact that both $\tilde{f}_s(v)$ and $\tilde{W}_s(v)$ are zero for

$v \to \infty$ we find

$$\int_v^\infty v' f_{in}(v')dv' = \zeta \nu_p v_T^2 \left(\frac{\tilde{W}_s(v)}{2v} + \frac{v_T \tilde{f}_s(v)}{v^2} \right) . \tag{26}$$

This expression can be simplified if we note that the left hand side can be expressed in terms of $\tilde{f}_{coll}(v)$ by integrating (12) over x and v, which gives

$$\int_v^\infty v' f_{in}(v')dv' = \nu(v) v \tilde{f}_{coll}(v) . \tag{27}$$

We combine these expressions to arrive at

$$\tilde{W}_s(v) = \frac{2v_T}{v} \left(\tilde{f}_{coll}(v) - \tilde{f}_s(v) \right) , \tag{28}$$

for the integrated wave distribution. Finally, we have for the integrated energy density of plasma waves

$$\tilde{\varepsilon}_w = \kappa T \int \tilde{W}(v) dv = 2\kappa T \int \frac{v_T}{v} \left(\tilde{f}_{coll}(v) - \tilde{f}_s(v) \right) dv . \tag{29}$$

The function $\tilde{f}_{coll}(v)$ is known provided the input function, $f_{in}(v)$, is known. For the illustrative example of equation (15) with the aide of equation (13) we find

$$\tilde{f}_{coll}(v) = \frac{4\Gamma(\delta - 1)}{\sqrt{\pi}\Gamma(\delta - \frac{1}{2})} \left(\frac{v}{v_s} \right)^2 \frac{n_s \lambda(v_s)}{v_s [1 + (v/v_s)^2]^{\delta - 1}} , \tag{30}$$

which is depicted in figure (2).

Evaluation of $\tilde{f}_s(v)$, on the other hand, will require solving the complete set of equations including both the background and the suprathermal particles. As in part B above, equation (25) can be integrated leading to a solution similar to that in equation (19) but with the integration constant ξ still unknown. However, we can set an upper limit on the plasma wave energy density assuming $\tilde{f}_s \ll \tilde{f}_{coll}$. Substituting equation (30) in equation (29) and integrating we then find

$$\tilde{\varepsilon}_w = \frac{4\Gamma(\delta - 2)}{\sqrt{\pi}\Gamma(\delta - \frac{1}{2})} n_s \kappa T \left(\frac{v_T}{v_s} \right) \lambda(v_s) \tag{31}$$

for the integrated plasma wave energy density.

The quantity $\tilde{\varepsilon}_w$ is the energy density of waves integrated along the field lines. This multiplied by the cross-sectional area perpendicular to the field line will give the total energy in waves. For comparison with the result in part B above we note that a significant level of Langmuir waves will be generated over a length of about $\lambda(v_s)$ so that an estimate of the mean energy density of the plasma waves is given by

$$\bar{\varepsilon}_w \approx \frac{\tilde{\varepsilon}_w}{\lambda(v_s)} \approx n_s \kappa T \left(\frac{v_T}{v_s} \right) . \tag{32}$$

Comparison of this with equation (23) shows that $\xi \approx (v_c/v_s)^2$ as stated there.

We emphasize that equation (31) is strictly an upper limit for the integrated plasma wave energy density. However, the value of $\tilde{f}_s(v)$ is needed for a more exact integration of equation (29). In figure (1) we saw that when considering collisions a hump is formed in the electron number distribution and that the velocity of the peak v_{peak} of the hump increases with depth (dashed line on the right). Langmuir wave growth ($\partial f/\partial v > 0$) is over the range of phase velocities v_c to v_{peak}. One expects that quasi-linear relaxation will cause the distribution to flatten out in this range, however, the level of the plateau is not known. One may assume that its level is determined by the conservation of the number of fast particles and use the solution f_{coll} to find the height of the plateau (McClements, Emslie and Brown 1985). This is not a correct proceedure as it will lead to a negative energy density of plasma waves. This can be seen by noting that if $\int(\tilde{f}_{coll} - \tilde{f}_s)dv = 0$ then $\tilde{\varepsilon}_w$ as expressed by equation (29) will be negative. Clearly the level must be below this. As mentioned above a numerical solution is neccessary to determine the number distribution of electrons and the exact energy density in plasma waves.

We can obtain an estimate for this level if as before we set $W_s(v_c) = 0$ for some v_c. Clearly the critical velocity v_c is a function of depth x, but if we assume that for some mean value, $\bar{v}_c$, the integrated wave distribution $\tilde{W}_s(\bar{v}_c) = 0$, then, from equation (28), we obtain $\tilde{f}_s(\bar{v}_c) = \tilde{f}_{coll}(\bar{v}_c)$. This combined with equation (25) and with the assumption that $\eta \ll 1$ indicates that $\int \tilde{f}_s \, dv \ll \int \tilde{f}_{coll} \, dv$ and that the contribution of $\tilde{f}_s$ to $\tilde{\varepsilon}_w$ in equation (29) can be ignored so that our upper limit is very close to the true value. A schematic plot of $\tilde{f}_s$ is given in figure (2) for the purpose of qualitative comparision with $\tilde{f}_{coll}(v)$. Eventhough the difference between $\tilde{f}_{coll}$ and $\tilde{f}_s$ is significant, it is unlikely that this difference can give rise to a discernable observable signiture. For example the bremsstrahlung x-rays from $\tilde{f}_{coll}$ and $\tilde{f}_s$ will have indentical spectra except at energies less than $E_s = \frac{1}{2}mv_s^2$. The required number of nonthermal electrons, however, will be larger by $\sim (v_s/v_T)^{1/2}$, which is the difference between the two curves at large velocities.

We also note from equation (28) that $\tilde{W}_s(v)$ increases with v so that most of the contribution to $\tilde{\varepsilon}_w$ comes from high velocities $v \sim v_s$, i.e. smaller wave vectors $k \sim \nu_p/v_s$. Then the more realistic value for $\Delta k/k$ is unity not $v_s/2v_T$ as was assumed by Emslie and Smith (1984) in their order of magnitude estimation. This will be even more clear in the last section where we examine the conversion of the Langmuir waves to transverse electromagnetic radiation.

Finally we note that the energy of the waves is a small fraction of energy of the high energy particles with total energy $\tilde{\varepsilon}_{tot} = \int \frac{1}{2}mv^2 \tilde{f}_{coll}(v)dv$: $\tilde{\varepsilon}_w \approx \tilde{\varepsilon}_{tot}(\kappa T/E_s)^{3/2} L(\delta)$, where $E_s = \frac{1}{2}mv_s^2$ and $L(\delta)$ is a function of δ of order unity (for the example given here, $L(\delta) = \sqrt{\frac{32}{9\pi}}\Gamma(\delta - 2)/\Gamma(\delta - \frac{7}{2}) \approx (\delta/2.5)^{3/2}$). The factor $(kT/E_s)^{3/2} = 2^{3/2}(v_T/v_s)^3$ is due to the fact that the collisional damping rate of the waves is higher than the collisional loss rate of the nonthermal particles by the factor $(v_s/v_T)^3$. Thus, even though the terms on the right sides of equations (1), (10), (16) or (24) are comparable, the energy density in the waves is negligible as compared to the energy density in the nonthermal particles. Furthermore, assuming that the energy density of the nonthermal particles is less than the energy density in the background plasma (otherwise, our assumption of a steady state homogeneous background plasma breaks down completely), then $\tilde{\varepsilon}_w \ll \varepsilon_T = n_T kT$.

Furthermore, as expected, the effects of the waves on the nonthermal electrons is comparable or smaller than the effect of the Coulomb collisions since the Coulomb collisions are the source of the plasma waves in the first place. Thus, the type of analysis such as Leach and Petrosian (1981) which neglect the presence of the waves will still give reliable estimates of the direct radiation signature of the nonthermal electrons. Moreover, if one includes the pitch angle diffusion terms, which tend to isotropize the distribution of the electrons, especially those at lower energies, the effects of the plasma waves will be further reduced.

III. THE RADIATION SIGNATURE OF THE WAVES

We now examine the observational consequences of the Langmuir wave generation discussed in the preceding section.

Even though the fraction of the total nonthermal energy converted into plasma waves, $R_w = \tilde{\varepsilon}_w/\tilde{\varepsilon}_{tot}$, is small, it can be larger than the fraction of energy radiated through Coulomb bremsstrahlung or synchrotron processes. Thus if a significant part of the plasma wave energy is converted to transverse electromagnetic waves (radiation for short), it could exceed the direct radiation from the nonthermal electrons.

A. *Direct Bremsstrahlung and Synchrotron Radiation*

For the purpose of comparison with emission by waves, we first give the expressions for direct radiation by the nonthermal and nonrelativistic electrons.

The total bremsstrahlung radiation can be obtained from the bremsstrahlung yield of the thick target model (cf., e.g. Petrosian 1973. Note the present $\ln\Lambda$ is one half of the $\ln\Lambda$ used in the earlier paper.) For a particle with initial energy E this yield is $Y_{brem}(E) = (4\alpha/3\pi\ln\Lambda)(E/mc^2)$, where α is the fine structure constant. For the distribution of equation (15) with characteristic energy $E_s = \frac{1}{2}mv_s^2$, the rate of energy produced in bremsstrahlung radiation by the injected beam is

$$\dot{\tilde{\varepsilon}}_{brem} = \frac{\Gamma(\delta-3)}{\sqrt{\pi}\Gamma(\delta-\frac{1}{2})}\left(\frac{8\alpha}{3\pi\ln\Lambda}\right)\left(\frac{E_s}{mc^2}\right)n_s v_s E_s \ . \tag{33}$$

Similarly, we can evaluate the rate of energy produced in synchrotron radiation $\dot{\bar{\varepsilon}}_{synch}$ from the total yield of a nonrelativistic particle with initial energy E:

$$Y_{synch}(E) = \left(\frac{2^{3/2}}{15\pi \ln \Lambda}\right)\left(\frac{B_\perp^2}{n_T mc^2}\right)\left(\frac{E}{mc^2}\right)^{3/2} \quad . \tag{34}$$

So that for the distribution used here, using equations (8) and (14), we can write

$$\dot{\bar{\varepsilon}}_{synch} = \frac{1}{2}\frac{\Gamma(\delta - \frac{7}{2})}{\Gamma(\delta - \frac{1}{2})} r_o^2 n_s v_s^2 B_\perp^2 \lambda(v_s)/c \ , \tag{35}$$

where $B_\perp$ is the component of the field perpendicular to the line of sight. Note that the ratio of these two rates is $\dot{\bar{\varepsilon}}_{synch}/\dot{\bar{\varepsilon}}_{brem} \approx 20(\nu_B/\nu_p)^2(E_s/20\text{keV})^{1/2}$. We have assumed here that the source is transparent to all radiation which may not be true. We will return to the optically thick cases at the end of this section.

B. *Radiation Produced by Waves*

We consider here two mechanisms for conversion of the plasma waves to radiation: 1) inelastic scattering of waves by electrons, and 2) wave-wave fusion into radiation with frequency $2\nu_p$. It appears that Zeitsev and Kaplan (1968) had the first of these processes in mind but their estimate of the spectrum or intensity of the radiation seems to be incorrect. As we shall see below, this mechanism is negligible as compared with the second one, which is the mechanism used by Emslie and Smith (1984). We use the expression for the rate of these processes given in Kaplan and Tsytovich (1973) (KT for short). We assume the weak magnetic field cases as we have done in the previous section.

1. *Electron Scattering*

The scattering of plasma waves (of frequency ν_p and wave vector k) by electrons of velocity v can produce radiation with frequency $\nu \leq kv/2\pi$ (cf. equation A-5 of KT). Transverse waves with frequency less than the plasma frequency do not propagate, so only electrons with velocities $v > 2\pi\nu_p/k$ are significant. Since the plasma waves generated by the plateau formation have wave vectors in the range $\nu_p/v_c > k/2\pi > \nu_p/v_s$ (with greater concentration towards the lower end) the frequency of radiation generated by electrons of velocity v will be $\leq \nu_p(v/v_s)$. Therefore only electrons with $v > v_s$ are of importance, so that the inelastic scattering by thermal electrons considered by Zeitsev and Kaplan (1968) produces transverse electromagnetic waves of frequency less than the plasma frequency which cannot propagate. An upper limit on the radiation produced can then be obtained by assuming that all of the nonthermal electrons produce radiation above the plasma frequency. The total radiation energy production rate of electron scattering then can be estimated to be

$$\dot{\tilde{\varepsilon}}_{es} \sim \pi r_o^2 n_s v_s^2 \tilde{\varepsilon}_w / c \ . \tag{36}$$

On the other hand, the synchrotron radiation produced by the nonthermal particles of velocity v_s in a magnetic field of strength B and energy density $\varepsilon_B = B^2/8\pi$ is given by equation (36), so that

$$\dot{\tilde{\varepsilon}}_{es}/\dot{\tilde{\varepsilon}}_{synch} \approx \bar{\varepsilon}_w/\varepsilon_B \ll 1 \ . \tag{37}$$

Thus electron scattering can be neglected since for most plasmas $\varepsilon_B \geq \varepsilon_T = n_T kT$ and $\bar{\varepsilon}_w < n_s kT \ll \varepsilon_T$.

More accurately, if we integratge equation (A-5) of KT over the distribution of waves and the nonthermal particles, we can obtain the total radiation produced from the scattering of the nonthermal electrons by the plasma waves. The integrated value requires knowing $(f\tilde{W})$. We approximate this quantity by $\tilde{f}\tilde{W}/\lambda(v_s)$ to obtain

$$\dot{\tilde{\varepsilon}}_{es} = \frac{8\pi}{5} \frac{\Gamma(\delta - \frac{7}{2})}{\Gamma(\delta - \frac{1}{2})} r_o^2 n_s v_s^2 \bar{\varepsilon}_w \lambda(v_s)/c \ , \tag{38}$$

which when compared with equation (35) gives

$$\dot{\tilde{\varepsilon}}_{es}/\dot{\tilde{\varepsilon}}_{synch} = \frac{2}{5}\left(\frac{8\pi\bar{\varepsilon}_w}{B_\perp^2}\right) \ll 1 \ . \tag{39}$$

2. *Wave-Wave Fusion*

The non-linear conversion of two plasma waves into electromagnetic radiation with frequency twice the plasma frequency can be much more significant. Using the appropriate equation form KT (cf. Table VI of the Appendix), one can estimate the total rate of energy production by this process (for $\Delta k \sim k \simeq 2\pi\nu_p/v_s$) to be

$$\dot{\bar{\varepsilon}}_{2\nu_p} \sim \frac{8\sqrt{3}}{5} r_o^2 c\lambda(v_s)\left(\frac{n_T}{mc^2}\right)\left(\frac{\bar{\varepsilon}_w^2 v_s^3}{\nu_p^3}\right) \ , \tag{40}$$

so that with the help of equations (32) and (36) we find

$$\dot{\bar{\varepsilon}}_{2\nu_p}/\dot{\bar{\varepsilon}}_{es} \approx n_T v_T^3/\nu_p^3 \approx 10^2 N_D \gg 1 \ . \tag{41}$$

Using our distribuiton of wave vectors (equation 28), we can obtain a more accurate value of $\dot{\bar{\varepsilon}}_{2\nu_p}$. The rate of energy production is (Table VI of KT)

$$\dot{\varepsilon}_{2\nu_p} = \frac{8\sqrt{3}\pi^2}{5}\frac{\nu_p(\kappa T)^2}{n_T mc^5}\int v^4 W^2(v)dv \ . \tag{42}$$

Using $kv_s = 2\pi\nu_p$ to rewrite this as an integral over wave vectors, we find that the energy produced is proportional to $\int W^2(k)\, k^{-6}\, dk \propto \int k^{-8} dk$, where equation (28) was used for the last relation. Therefore the rate of fusion of plasma waves is strongly peaked at low k values, so that the effective spread of wave vectors is not given by $k_{max} - k_{min}$, but is on the order of $k_{min}(\sim 2\pi\nu_p/v_s)$. Finally, we find that the integrated energy production rate due to the fusion of two plasma waves is (approximating the Γ functions by the Sterling formula)

$$\dot{\bar{\varepsilon}}_{2\nu_p} = \left(\frac{3.8}{\delta}\right)^{3/2} r_o^2 c\lambda(v_s)\left(\frac{n_T}{mc^2}\right)\left(\frac{\bar{\varepsilon}_w^2 v_s^3}{\nu_p^3}\right) \ , \tag{43}$$

where we have approximated $\tilde{W}^2$ by $\tilde{W}\tilde{W}/\lambda(v_s)$. The ratio of this rate to the synchrotron energy production rate is (approximating the Γ functions by the Sterling formula)

$$\dot{\bar{\varepsilon}}_{2\nu_p}/\dot{\bar{\varepsilon}}_{synch} = 20\left(\frac{8\pi\bar{\varepsilon}_w}{B_\perp^2}\right)N_D \ , \tag{44}$$

As we have stated before $\bar{\varepsilon}_w \ll B_\perp^2$, but since $N_D \gg 1$, this ratio may or may not exceed unity. Using equation (32) this ratio can be written

$$\dot{\varepsilon}_{2\nu_p}/\dot{\varepsilon}_{synch} = 10^2 \left(\frac{4}{\delta}\right)^{3/2} \left(\frac{n_T}{10^{10}\,\mathrm{cm}^{-3}}\right)^{1/2} \left(\frac{300\mathrm{G}}{B_\perp}\right)^2 \left(\frac{T}{10^6\mathrm{K}}\right)^{7/2} \left(\frac{50\mathrm{keV}}{E_s}\right) \left(\frac{n_s v_s}{n_T v_T}\right) . \tag{45}$$

In general we expect the last term to be less than one otherwise the drift velocity of the reverse current will exceed the thermal velocity and therefore destroy the beam rapidly. If $(n_s v_s / n_T v_T) < .01$, then for coronal conditions this ratio is $\lesssim 1$, but it will be much less than one in the chromosphere and the photosphere. Note that our estimate of the yield of $2\nu_p$ photons is lower than that by Emslie and Smith (1984) because, as mentioned above, they over estimate the wave energy density by $v_s/2v_T$ and the $2\nu_p$ emission is proportional to the square of this quantity, which could be as much as 10^3.

Most importantly, however, since in most astrophysical situations $\nu_B \sim \nu_p$ and that, in general, the first few harmonics of synchrotron radiation are self-absorbed, it is unlikely that the $2\nu_p$ photon can escape. In the case of solar flares it is clear already from equations (33) and (35) that the syncrotron yield is $\approx 10^{-1}$ the bremsstrahlung yield. This means that optically thin synchrotron emission gives microwave energy flux comparable to the bremsstrahlung x-ray energy flux while as was pointed out as soon as x-ray observations of the impulsive phase of flares were available (Peterson and Winckler 1959) the observed microwave flux is negligible compared to the x-ray flux. This problem can be resolved by inclusion of self-absorption of the synchrotron radiation which reduces the flux by the large amount necessary if self-absorption occurs up to high harmonics $\nu \approx 10\nu_B$ (Holt and Ramaty, 1969). Thus, if $10\nu_B > 2\nu_p$, or equivalently if $B > 6.4\ (n_T/10^8\mathrm{cm}^{-3})^{1/2}$ gauss, the $2\nu_p$ photons will also be absorbed and unobservable.

Finally, we note that $\dot{\varepsilon}_{es} \ll \dot{\varepsilon}_{2\nu_p} \ll \gamma_{coll}\bar{\varepsilon}_w$ where the last term is the rate of damping of the waves by elastic collisions and is the term included in our equation. This justifies the neglect of the inelastic process in the kinetic equation for the waves.

IV. SUMMARY

We have solved the coupled kinetic equations of nonthermal electrons and estimated the level of the plasma waves produced by the bump-in-tail instability which arises as a result of the Coulomb collisions of the nonthermal electrons with the (much colder) background plasma. We have also investigated the radiation signature of these waves coming to the following conclusions.

1) For a spatially homogeneous (but perhaps unrealistic) electron beam situation the earlier estimation by Zaitsev and Kaplan (1968) is correct if the spread in the range of the wave vectors satisfies the relation $\Delta k/k \sim 1$. We note here that Emslie and Smith (1984) incorrectly use $\Delta k/k \sim v_s/2v_T$ and therefore overestimate the wave energy density leading to a large error in the expected amount of radiation produced by fusion into electromagnetic waves at twice the plasma frequency.

2) For the more realistic spatially inhomogeneous beam one can put an upper limit on the total (spatially integrated) wave spectrum and energy density. This clarifies some of the ambiguities which arise in trying to estimate the wave energy density for the homogeneous case.

3) Since the Coulomb collisions are the primary agent for the production of plasma waves, the effects of the interaction of plasma waves with the nonthermal electrons could be at most as important as the effects of collisions. Therefore we expect the wave-particle interactions to have a significant, but not dominant, effect on the overall distribution of the electrons. It is unlikely that such effects can be discerned in the observed x-ray spectrum of the nonthermal electrons.

4) Eventhough a significant fraction of the energy of the nonthermal particles is transmitted to plasma waves, the plasma wave energy density is much smaller than the nonthermal particle energy density because the collisional damping of the waves by the background plasma is much faster than the collisional energy loss rate of the nonthermal particles. Thus the plasma waves act as agents for transferring the energy of the nonthermal electrons to

the background plasma.

5) Because of the low energy density of the plasma waves we find that the intensity of the radiation obtained by conversion of these waves to transverse electromagnetic waves to be much smaller than that estimated by Zaitsev and Kaplan (1968) or Emslie and Smith (1984). In particular the radiation (microwave) yield due to inelastic scattering of the waves by electrons is expected to occur mainly below the plasma frequency and down from the direct production of microwave radiation via synchrotron (or cyclotron) emission by the ratio of magnetic field energy density to the plasma wave energy density which we show to be a large number.

6) A more efficient process of emission of plasma waves to transverse electromagnetic waves is the fusion of two Langmuir waves into transverse electromagnetic radiation at twice the plasma frequency. We show that even the level of this radiation is less or comparable to the direct synchrotron radiation. More importantly, both of these processes lead to radiation levels which will greatly exceed the observed levels if the synchrotron absorption is ignored. Even at modest values of the magnetic field this cannot be justified so that as long as $B > 6.4\ (n/10^8\mathrm{cm}^{-3})^{1/2}$ gauss the absorption is large enough to prevent an observable signature of the plasma waves produced by the bump-on-tail instability considered here.

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APPENDIX

Although the distributions of electrons and of Langmuir waves will in general depend on three spacial dimensions and the three velocity components, the situation of an electron beam injected into a background thermal plasma in the presence of a weak magnetic field introduces conditions which will allow us to reduce our description of the electron and wave distributions to a dependence on the distance along the magnetic field line (x in the paper) and the component of velocity along the magnetic field $v_{\parallel}$. We will arrive at equations describing the evolution of the one-dimensional distributions (in this appendix we will suppress the spatial dependence and focus on the velocity dependence) $\hat{f}(v_{\parallel})$ and $\hat{W}(v_{\parallel})$ obtained from integrating the full distributions for $f(\mathbf{v})$ and $W(\mathbf{k})$ as

$$\hat{f}(v_{\parallel}) = 2\pi \int f(\mathbf{v}) v_{\perp} dv_{\perp} \quad , \tag{A1}$$

$$\hat{W}(k_{\parallel}) = 2\pi \int W(\mathbf{k}) k_{\perp} \frac{dk_{\perp}}{(2\pi)^3} \quad . \tag{A2}$$

In part *a* we derrive the Coulomb collision terms and in part *b* the wave-particle interaction terms. The final results are equations (1) and (2) of the paper, where $f(v) = \hat{f}(v_{\parallel})$ and $W(v) = \left|\frac{dk_{\parallel}}{dv_{\parallel}}\right| \hat{W}(k_{\parallel})/\kappa T$ with $k_{\parallel} v_{\parallel} = \omega_p$ are the distributions used there.

a. *Coulomb Collisions*

The effects of collisions with the thermal particles on the electron distribution is evaluated using a Fokker-Plank analysis. The change in the distribution function is written as (cf. e.g. Krall and Trivelpiece 1973)

$$\left.\frac{\delta f}{\delta t}\right|_{coll} = -\sum_i \frac{\partial}{\partial v_i} \langle \frac{\Delta v_i}{\Delta t} \rangle f + \frac{1}{2} \sum_{i,k} \frac{\partial^2}{\partial v_i \partial v_k} \langle \frac{\Delta v_i \Delta v_k}{\Delta t} \rangle f \quad , \tag{A3}$$

where $f(\mathbf{v}) = f(v_x, v_y, v_z)$. For beam velocities much larger than the velocity of the thermal particles, the values of the average rate of changes in the velocities are given by

$$\langle \frac{\Delta v_i}{\Delta t} \rangle = -\sum_{\alpha} \frac{4\pi n_{\alpha} e^2 e_{\alpha}^2}{m \mu_{\alpha}} \ln \Lambda \quad , \quad \mu_{\alpha} = \frac{m m_{\alpha}}{m + m_{\alpha}} \tag{A4}$$

and

$$\langle\frac{\Delta v_i \Delta v_k}{\Delta t}\rangle = \sum_\alpha \frac{4\pi n_\alpha e^2 e_\alpha^2}{m^2}\frac{\ln\Lambda}{v}(\delta_{ik} - v_i v_k/v^2) \ , \tag{A5}$$

where α labels the particle type in the background plasma and δ_{ik} is the Kronecker delta. For a fully ionized hydrogen plasma we obtain

$$\left.\frac{\delta f}{\delta t}\right|_{coll} = 3C\sum_i \frac{\partial}{\partial v_i}\left(\frac{v_i}{v^3}f\right) + C\sum_{ik}\frac{\partial^2}{\partial v_i \partial v_k}\Big((\delta_{ik} - v_i v_k/v^2)\Big)f/v \ , \tag{A6}$$

where $C = 4\pi c^4 r_o^2 n_T \ln\Lambda$. Integrating this equation over $v_\perp$ we find

$$\left.\frac{\delta \hat{f}}{\delta t}\right|_{coll} = 2\pi C\int \frac{\partial}{\partial v_\parallel}\left(\frac{3v_\parallel^3}{v^5}f + \frac{v_\perp^2}{v^3}\frac{\partial f}{\partial v_\parallel}\right) v_\perp dv_\perp \ , \tag{A7}$$

where $f = f(v_\parallel, v_\perp)$. Assuming that $f(v_\parallel, v_\perp)$ has a width in $v_\perp$ of $\Delta v_\perp$, then for $(\Delta v_\perp/v_\parallel)^2 \ll 1$ we have

$$\left.\frac{\delta \hat{f}}{\delta t}\right|_{coll} = C\frac{\partial}{\partial v_\parallel}\left(\frac{3}{v_\parallel^2}\hat{f} - \frac{15}{2}\frac{1}{v_\parallel^2}\left(\frac{\Delta v_\perp}{v_\parallel}\right)^2\hat{f} + \frac{1}{v_\parallel}\left(\frac{\Delta v_\perp}{v_\parallel}\right)^2\frac{\partial \hat{f}}{\partial v_\parallel}\right) \ , \tag{A8}$$

where $2\pi\int_0^\infty (v_\perp/v_\parallel)^2 f v_\perp dv_\perp \approx (\Delta v_\perp/v_\parallel)^2\hat{f}$ was used.

For an injection of electrons which is strongly beamed $(\Delta v_\perp/v_\parallel)^2 \ll 1$, the first term will be much larger than the second term, which we neglect. The last term involves the derivative $\frac{\partial \hat{f}}{\partial v_\parallel}$. It will be important if its magnitude is comparable to that of the wave particle term $v_\parallel W\frac{\partial \hat{f}}{\partial v_\parallel}$ found below in part b. We will show that Langmuir wave genreration by the beam leads to $W \approx \varepsilon_w/v_\parallel kT \approx n_s v_T/v_\parallel^2$ (cf. equation 33). Therfore the last term can be neglected provided that

$$\left(\frac{\Delta v_\perp}{v_\parallel}\right)^2 \ll \frac{1}{4\eta} = \pi^2(n_s/n_T)(\nu_p/\nu(v_T)) \ , \tag{A9}$$

where n_s, n_T, ν_p, and $\nu(v_T)$ are the beam density, background density, plasma frequency, and collision frequency of thermal electrons, respectively. Typically $\eta < 1$ (see equation 18) therefore this condition will be satisfied. Therefore the collisions lead to a term,

$$\left.\frac{\delta \hat{f}}{\delta t}\right|_{coll} = C\frac{\partial}{\partial v_\parallel}\left(\frac{3}{v_\parallel^2}\hat{f}\right) \ . \tag{A10}$$

The Landau damping of waves with wave vectors $k < \omega_p/v_T$ is negligible, so that collisional damping (free-free absorbtion) must be considered. For Langmuir waves the damping rate $\gamma_{coll} = \frac{\omega_p^4 \ln \Lambda}{16\pi n_e v_e^3}$ (Ginzburg 1961) is independent of the wave vector so that the integration over $k_\perp$ can be carried out simply leading to the damping term $-\gamma_{coll} W$ on the right hand side of equation (2).

b. *Wave-Particle Interactions*

The evolution of the distribution of particles, $f(\mathbf{p})$, and waves of mode σ, $N^\sigma(\mathbf{k})$, due to the emission and absorption of waves by the particles is determined by the quasi-linear equations. In the classical limit these are (Melrose 1980)

$$\frac{df}{dt} = \int \frac{d^3k}{(2\pi)^3}\, \hbar\mathbf{k} \cdot \frac{\partial}{\partial\mathbf{p}} \left(w^\sigma(\mathbf{p},\mathbf{k})(f + N^\sigma \hbar\mathbf{k} \cdot \frac{\partial f}{\partial \mathbf{p}}) \right) \quad , \tag{A11}$$

and

$$\frac{dN^\sigma}{dt} = \int d^3p\, w^\sigma(\mathbf{p},\mathbf{k})(f + N^\sigma \hbar\mathbf{k} \cdot \frac{\partial f}{\partial \mathbf{p}}) \quad , \tag{A12}$$

where $w^\sigma(\mathbf{p},\mathbf{k})$ is the probability per unit time that a particle having momentum $\mathbf{p}$ will emit a σ-wave with wave vector $\mathbf{k}$ in the range $\frac{d^3k}{(2\pi)^3}$.

For Langmuir waves

$$w^l(\mathbf{p},\mathbf{k}) = \frac{4\pi^2 e^2 \omega_p}{\hbar k^2}\, \delta(\omega_p - \mathbf{k}\cdot\mathbf{v}) \quad , \tag{A13}$$

and their energy density per $\frac{d^3k}{(2\pi)^3}$ is $W = \hbar\omega_p N^l$, so that equations (A11) and (A12) become

$$\frac{df}{dt} = \frac{\partial}{\partial\mathbf{v}} \cdot \left(\mathbf{D} \cdot \frac{\partial f}{\partial \mathbf{v}} + \mathbf{A} f \right) \tag{A14}$$

and

$$\frac{dW}{dt} = \gamma_k W + \alpha_k \quad , \tag{A15}$$

with

$$\mathbf{D} = (\frac{e^2}{2\pi m^2}) \int d^3k\, \hat{\mathbf{k}}\, \hat{\mathbf{k}}\, W\, \delta(\omega_p - \mathbf{k}\cdot\mathbf{v}) \tag{A16}$$

$$\mathbf{A} = \frac{e^2\omega_p}{2\pi m}\int d^3k\,\frac{\hat{\mathbf{k}}}{k}\,\delta(\omega_p - \mathbf{k}\cdot\mathbf{v}) \tag{A17}$$

$$\gamma_k = \frac{4\pi^2 e^2\omega_p}{k^2 m}\int d^3v\,\mathbf{k}\cdot\frac{\partial f}{\partial \mathbf{v}}\,\delta(\omega_p - \mathbf{k}\cdot\mathbf{v}) \tag{A18}$$

$$\alpha_k = \left(\frac{2\pi e\omega_p}{k}\right)^2\int d^3v\,f\,\delta(\omega_p - \mathbf{k}\cdot\mathbf{v})\ , \tag{A19}$$

where we have changed variables ($f(\mathbf{p})d^3p = f(\mathbf{v})d^3v$) and taken the nonrelativistic limit ($\mathbf{p} = m\mathbf{v}$).

We will now derive one-dimensional forms of the above equations and give the restrictions on the electron distributions for which these equations hold. Integrating equation ($A19$) over v_x, the component of velocity along the magnetic field,

$$\alpha_k = \frac{4\pi^2 e^2\omega_p^2}{k_\parallel k^2}\int dv_y dv_z\,f\left(\frac{\omega_p}{k_\parallel} + \frac{k_y v_y}{k_\parallel} + \frac{k_z v_z}{k_\parallel}, v_y, v_z\right)\ . \tag{A20}$$

Using the Taylor expansion

$$f\left(v_\parallel + \frac{k_y v_y}{k_\parallel} + \frac{k_z v_z}{k_\parallel}, v_y, v_z\right) = f(v_\parallel, v_y, v_z) + \sum_{n=1}^{\infty}\frac{\left(\frac{k_y}{k_\parallel}v_y + \frac{k_z}{k_\parallel}v_z\right)^n}{n!}\frac{\partial^n f(v_\parallel, v_y, v_z)}{\partial v_\parallel^n}\ , \tag{A21}$$

where $v_\parallel = \omega_p/k_\parallel$, keeping the first two terms in the sum, we obtain

$$\alpha_k = \frac{4\pi^2 e^2\omega_p^2}{k_\parallel k^2}\left(\hat{f}(v_\parallel) + \frac{(\Delta v_\perp)^2}{4}\left(\frac{k_\perp}{k_\parallel}\right)^2\frac{\partial^2\hat{f}(v_\parallel)}{\partial v_\parallel^2}\right)\ . \tag{A22}$$

To arrive at equation ($A22$) we have used $\int v_y f(v_\parallel, v_y, v_z)dv_y dv_z = 0$ and made the approximation $\int v_y^2 f(v_\parallel, v_y, v_z)dv_y dv_z = \frac{1}{2}(\Delta v_\perp)^2\hat{f}(v_\parallel)$ (similar relations were used for the v_z integrals). Now if we assume that the electrons are beamed, $(\frac{\Delta v_\perp}{v_\parallel})^2 \ll 1$ and $(\frac{\Delta k_\perp}{k_\parallel})^2 \ll 1$ then we can neglect the second term in equation ($A22$).

Writing out equation ($A18$) we have

$$\gamma_k = \frac{4\pi^2 e^2\omega_p}{k^2 m}\int d^3v\left(k_\parallel\frac{\partial f}{\partial v_x} + k_y\frac{\partial f}{\partial v_y} + k_z\frac{\partial f}{\partial v_z}\right)\delta(\omega_p - k_\parallel v_x - k_y v_y - k_z v_z)\ , \tag{A23}$$

where $f = f(v_x, v_y, v_z)$ and $\mathbf{k} = (k_\parallel, k_y, k_z)$. This expression involves two types of terms. The first term we encountered above when finding α_k. The second type occurs in the last two terms, which are similar due to the symmetry between the y and z directions. Integration of the last terms is done using

$$\int dx \frac{\partial f(x,y,z)}{\partial y}\delta(\omega_p - a_x x - a_y y - a_z z) = \frac{1}{a_x}\left(\frac{\partial}{\partial y} - \frac{a_y}{a_x}\frac{\partial}{\partial x_\parallel}\right) f(x_\parallel + \frac{a_y}{a_x}y + \frac{a_z}{a_x}z, y, z) \ , \tag{A24}$$

where $x_\parallel = \frac{\omega_p}{a_x}$. Integration of the first term over dv_y is zero. Then making the same approximations used above for the second terms gives

$$\gamma_k = \frac{4\pi^2 e^2 \omega_p}{k^2 m}[1 - (\frac{k_\perp}{k_\parallel})^2]\frac{\partial \hat{f}(v_\parallel)}{\partial v_\parallel} \ . \tag{A25}$$

Multiplication of equation ($A15$) by $2\pi k_\perp$ and integrating over $\frac{d^3k}{(2\pi)^3}$ gives

$$\frac{d\hat{W}(v_\parallel)}{dt} = \frac{\pi\omega_p}{n_T}v_\parallel^2\frac{\partial \hat{f}(v_\parallel)}{\partial v_\parallel}\hat{W}(v_\parallel)\left(1 - 2(\frac{\Delta k_\perp}{k_\parallel})^2\right) + \frac{\omega_p^3 m}{4\pi n_T}v_\parallel \ln(v_\parallel/v_T)\hat{f}(v_\parallel) \ , \tag{A26}$$

where $2\pi\int k_\perp^2 \frac{W(\mathbf{k})k_\perp dk_\perp}{(2\pi)^3} = (\Delta k_\perp)^2\hat{W}(k_\parallel)$ was used. The assumption that $(\Delta k_\perp/k_\parallel)^2 \ll 1$ in equation ($A26$) leads to the wave-particle terms in equation (2) of the paper. Note that $\omega_p = k_\parallel v_\parallel$ has been used throughout.

Now we outline the proceedure used to obtain the wave-particle terms in equation (1). Since many of the same approximations used above are used here we will omit most of the details. Notice that $\mathbf{A}$ simplifies immediately to

$$\mathbf{A} = \frac{e^2\omega_p}{2\pi m}\int \hat{\mathbf{k}}\, k\, dk\, d\cos\chi\, d\eta\, \delta(\omega_p - kv\cos\chi) \ , \tag{A27}$$

where $\mathbf{k}\cdot\mathbf{v} = kv\cos\chi$. Integration over η eliminates the components perpendicular to $\mathbf{v}$, so,

$$\mathbf{A} = \frac{\mathbf{v}}{v}\frac{e^2\omega_p}{m}\int_0^{\omega_p/v_T} kdk\int_{-1}^{1}\cos\chi\delta(\omega_p - kv\cos\chi)d\cos\chi = \frac{e^2\omega_p^2}{m}\frac{\mathbf{v}}{v^3}\int_{\omega_p/v}^{\omega_p/v_T}\frac{dk}{k} \ , \tag{A28}$$

where for $k > \omega_p/v_T$ Langmuir waves are heavily Landau damped. Therefore

$$\mathbf{A} = \frac{e^2\omega_p^2}{m}\frac{\mathbf{v}}{v^3}\ln(v/v_T) \ . \tag{A29}$$

The reduction of equation ($A16$) follows similarly. We find

$$\frac{\partial}{\partial \mathbf{v}} \cdot \left(\mathbf{D} \cdot \frac{\partial f}{\partial \mathbf{v}} \right) = \left(\frac{2\pi e}{m} \right)^2 \frac{\partial}{\partial v_{\parallel}} \left(\frac{\hat{W}(k_{\parallel})}{v_{\parallel}} \frac{\partial f}{\partial v_{\parallel}} \right) , \tag{$A30$}$$

where the definition of the one-dimensional distribution $\hat{W}(k_{\parallel})$ was used.

Now we integrate equation (A14) over $v_{\perp}$ to obtain

$$\frac{d\hat{f}(v_{\parallel})}{dt} = \frac{\partial}{\partial v_{\parallel}} \left(\frac{\pi \omega_p^2}{m n_T} \frac{\hat{W}(v_{\parallel})}{v_{\parallel}} \frac{\partial \hat{f}(v_{\parallel})}{\partial v_{\parallel}} + \frac{\omega_p^4}{4\pi n_T} \frac{\ln(v_{\parallel}/v_T)}{v_{\parallel}^2} \hat{f}(v_{\parallel}) \right) . \tag{$A31$}$$

The first term follows directly from equation (A30). The second term is obtained from integration of $\mathbf{A}f$ over $v_{\perp}$, which is similar to the integration done in part a) of this appendix (cf. equations A7 and A8), therefore requires that $(\Delta v_{\perp}/v_{\parallel})^2 \ll 1$.

This then completes our description of the wave-particle effects, combining equations ($A31$), ($A26$), and ($A10$) we arrive at equations (1) and (2), the set of equations used in the paper. These equations are valid for electrons whose velocity along the field line is larger than the dispersion of the perpendicular velocity.

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FIGURE CAPTIONS

FIG. 1.—The electron number distribution is plotted vs. velocity for the input function given by equation (15) with $\delta = 4$, $n_s/n_T = 10^{-3}$, and $v_T/v_s = .04$. The curves are labeled by the ratio $x/\lambda(v_s)$ and the thermal distribution is also included. The dashed curves show the dependence of v_c and v_{peak} with depth.

FIG. 2.—The integrated (over spatial coordinate x) electron number distribution is plotted vs. velocity with the parameters set to the same values as in figure (1). The solid lines are the distribution at the injection point, the thermal distribution, and $\bar{f}_{coll}(v) = \tilde{f}_{coll}/\lambda(v_s)$. The dashed line is a schematic drawing of $\bar{f}_s(v) = \tilde{f}_s(v)/\lambda(v_s)$.

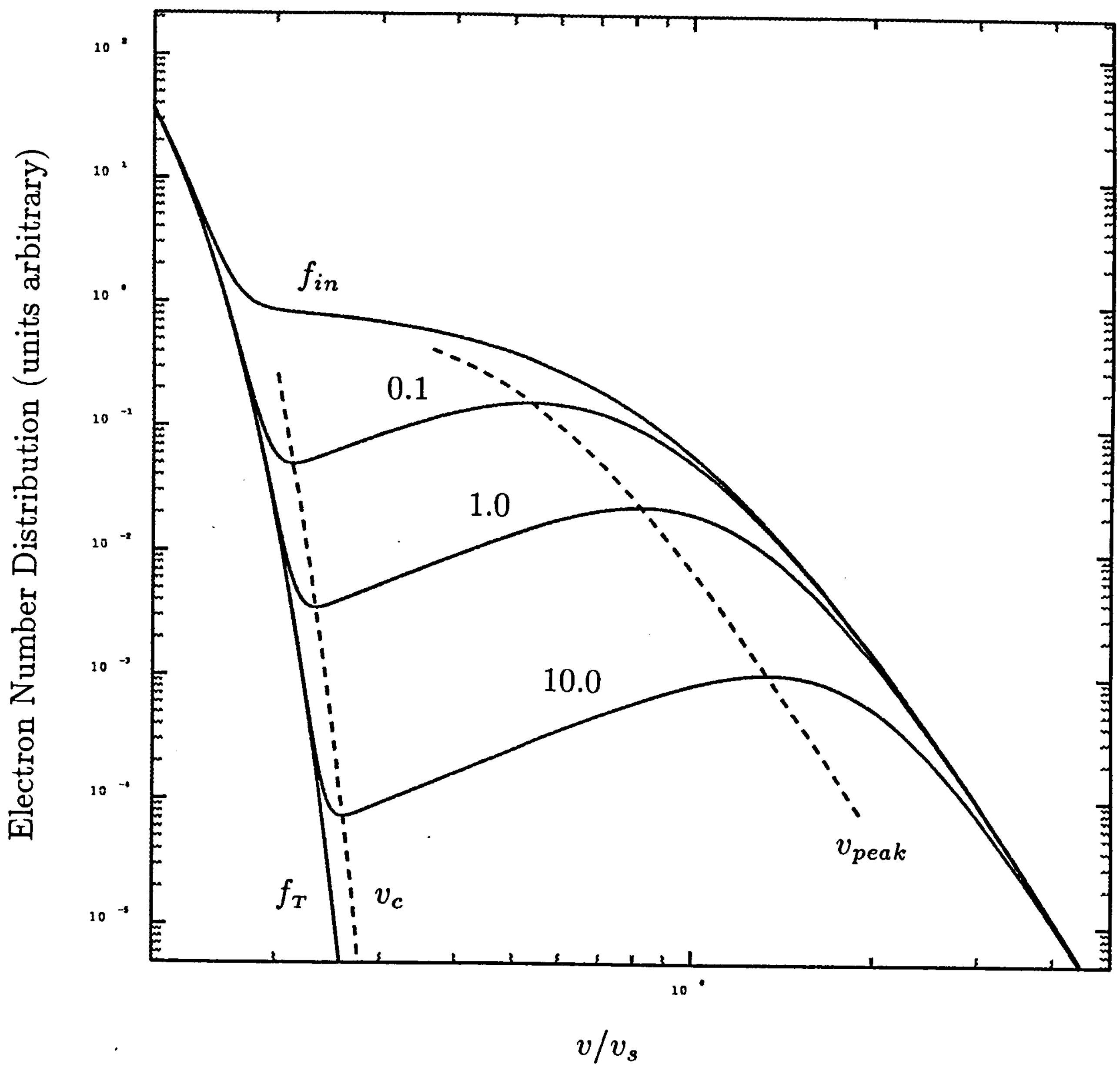
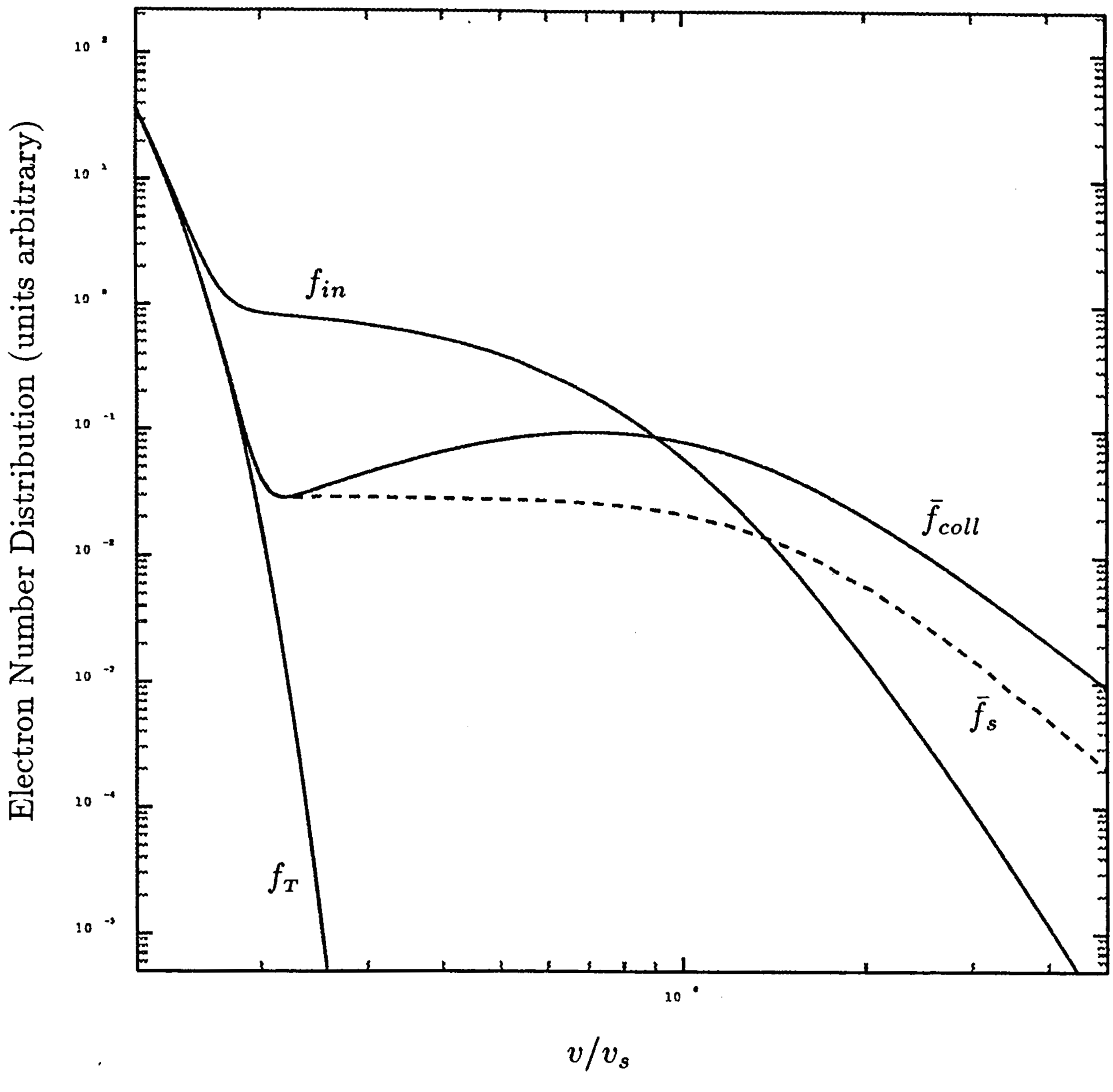

Electron Number Distribution (units arbitrary)
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
f_{in}
0.1
1.0
10.0
v_{peak}
f_T
v_c
10^{0}
v/v_s

Figure 1.



Electron Number Distribution (units arbitrary)
f_{in}
$\bar{f}_{coll}$
$\bar{f}_s$
f_T
v/v_s

Figure 2.

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